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The HMW effect in noncommutative quantum mechanics

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Abstract
The HMW effect in noncommutative quantum mechanics is studied. By solving the Dirac equations on noncommutative (NC) space and noncommutative phase space, we obtain topological HMW phase on NC space and NC phase space, respectively, where the additional terms related to the space–space and momentum–momentum noncommutativity are given explicitly.

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1. Introduction

The study of physics effects on noncommutative space has attracted much attention in recent years, because the effects of the space noncommutativity may become significant not only in the string scale but also at the very high energy level (Tev and higher energy level). Besides the field theory, there are many papers devoted to the study of various aspects of quantum mechanics on NC space with usual (commutative) time coordinate. For example, the topological AB and AC effects on NC space and even on NC phase space have been studied [1–6]. In this paper, we will deal with another very interesting topological effect, HMW effect, on NC space and NC phase space, respectively. The HMW effect was first discussed in 1993 by He and Meckellar [7] and a year later independently by Wilkens [8]. The HMW effect corresponds to a topological phase related to a neutral spin-$1/2$ particle with non-zero electric dipole moving in the magnetic field, and in 1998, Dowling, Willianms and Franson point out that the HMW effect can be partially tested using metastable hydrogen atoms [9]. Just as the AB, AC effects, the HMW effect has the same importance in the literature, and the study of the correction of the space (and momenta) noncommutativity to the HMW effect will be meaningful.

To begin with, let us first give a brief review of some properties of noncommutative space and noncommutative phase space. In NC space, the coordinate and momentum operators satisfy the following commutation relations (we set $\hbar = c = 1$ in this paper):

$$[\hat{x}_i, \hat{x}_j] = \mathrm{i}\theta_{ij}, \qquad [\hat{p}_i, \hat{p}_j] = 0, \qquad [\hat{x}_i, \hat{p}_j] = \mathrm{i}\delta_{ij}, \tag{1}$$

where $\hat{x}_i$ and $\hat{p}_i$ are the coordinate and momentum operators on a NC space. When a spin-1/2 particle moves in a electromagnetic field, the Dirac equation for the particle, usually, can be written as $[\mathrm{i}\gamma_\mu\partial^\mu + S_\mu\gamma^\mu - m]\psi = 0$, here S_μ is a Lorentz vector which depends not only on the electromagnetic field in which the particle moves but also on the electromagnetic properties of the particle itself. On the NC space, this Dirac equation becomes

$$[\mathrm{i}\gamma_\mu\partial^\mu + S_\mu\gamma^\mu - m] \star \psi = 0, \tag{2}$$

i.e., just replace normal product to a star product, then the Dirac equation in commuting space will change into the Dirac equation in NC space. The Moyal–Weyl (or star) product between two functions is defined by

$$(f * g)(x) = \mathrm{e}^{\frac{\mathrm{i}}{2}\theta_{ij}\partial_{x_i}\partial_{x_j}} f(x_i)g(x_j) = f(x)g(x) + \frac{\mathrm{i}}{2}\theta_{ij}\partial_i f \partial_j g\bigg|_{x_i=x_j} + \mathcal{O}(\theta^2), \tag{3}$$

where $f(x)$ and $g(x)$ are two arbitrary functions. Other than to solve the NC Dirac equation by using the star product, an equivalent method will be used in this paper, that is, we replace the star product in the Dirac equation with the usual product by shift, the NC coordinates defined in [2], i.e.,

$$\hat{x}_i = x_i - \tfrac{1}{2}\theta_{ij}p_j, \qquad \hat{p}_i = p_i, \tag{4}$$

as well as a shift for the vector S_μ,

$$S_\mu \to \hat{S}_\mu = S_\mu + \tfrac{1}{2}\theta^{\alpha\beta}p_\alpha\partial_\beta S_\mu. \tag{5}$$

Then, the Dirac equation can be solved in the commuting space and the noncommutative properties can be realized through the terms related to θ.

The Bose–Einstein statistics in noncommutative quantum mechanics requires both space–space and momentum–momentum noncommutativity, the space in this case is called NC phase space. On NC phase space, the commutation relations (1) should be replaced with

$$[\hat{x}_i, \hat{x}_j] = \mathrm{i}\theta_{ij}, \qquad [\hat{p}_i, \hat{p}_j] = \mathrm{i}\bar{\theta}_{ij}, \qquad [\hat{x}_i, \hat{p}_j] = \mathrm{i}\delta_{ij}. \tag{6}$$

and the star product in equation (2), on NC phase space, defines

$$\begin{aligned}(f * g)(x, p) &= \exp\left(\frac{\mathrm{i}}{2\alpha^2}\theta_{ij}\partial_i^x\partial_j^x + \frac{\mathrm{i}}{2\alpha^2}\bar{\theta}_{ij}\partial_i^p\partial_j^p\right) f(x, p)g(x, p)\\ &= f(x, p)g(x, p) + \frac{\mathrm{i}}{2\alpha^2}\theta_{ij}\partial_i^x f\partial_j^x g\bigg|_{x_i=x_j} + \frac{\mathrm{i}}{2\alpha^2}\bar{\theta}_{ij}\partial_i^p f\partial_j^p g\bigg|_{p_i=p_j} + \mathcal{O}(\theta^2),\end{aligned} \tag{7}$$

where $\mathcal{O}(\theta^2)$ stands for the second and higher order terms of θ and $\bar{\theta}$. To replace the star product in the Dirac equation on NC phase space we need a generalization of the shift in equation (4), i.e.,

$$x_\mu \to \alpha x_\mu - \frac{1}{2\alpha}\theta_{\mu\nu}p^\nu, \qquad p_\mu \to \alpha p_\mu + \frac{1}{2\alpha}\bar{\theta}_{\mu\nu}x^\nu, \tag{8}$$

and together with a shift

$$S_\mu \to \hat{S}_\mu = \alpha S_\mu + \frac{1}{2\alpha}\theta^{\alpha\beta}p_\alpha\partial_\beta S_\mu, \tag{9}$$

which is the partner of the shift in equation (5) on NC phase space.

2. Review of the HMW effect on (2 + 1)-dimensional commutative space time

In order to study the NC properties of the HMW effect, a brief review of the effect on (2 + 1)-dimensional commutative space time is necessary. The Lagrange of a spin-1/2 neutral particle with electric dipole μ_e moving in the electromagnetic field is given by

$$L = \bar{\psi}\mathrm{i}\gamma^{\mu}\partial_{\mu}\psi - m\bar{\psi}\psi - \mathrm{i}\tfrac{1}{2}\mu_e\bar{\psi}\sigma^{\mu\nu}\gamma_5\psi F_{\mu\nu}. \tag{10}$$

The last term in the Lagrangian represents the HMW effect. Using the identity $\sigma^{\mu\nu}\gamma_5 = (\mathrm{i}/2)\epsilon^{\mu\nu\alpha\beta}\sigma_{\alpha\beta}$, the Lagrangian becomes

$$L = \bar{\psi}\mathrm{i}\gamma^{\mu}\partial_{\mu}\psi - m\bar{\psi}\psi + \tfrac{1}{2}\mu_e\tilde{F}_{\mu\nu}\bar{\psi}\sigma^{\mu\nu}\psi, \tag{11}$$

where $\tilde{F}$ is the Hodge star of F, i.e. $\tilde{F}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}F^{\alpha\beta}$. Similar as AB, AC and other topological effects, the HMW effect is also usually studied in (2 + 1) dimension, because the particle moves in a plane.

We restrict the particle moves on a plane (say $(x - y)$-plane), then the problem can be treated in (2 + 1) space time. We use the conventions $g_{\mu\nu} = \mathrm{diag}(1, -1, -1)$ and the anti-symmetric tensor $\epsilon_{\mu\nu\alpha}$ with $\epsilon_{012} = +1$. We will use three 4×4 Dirac matrices which can describe spin up and down in the notional z-direction for a particle and its anti-particle [10]. In (2 + 1) dimensions, these Dirac matrices satisfy the following relation:

$$\gamma^{\mu}\gamma^{\nu} = g^{\mu\nu} - \mathrm{i}\gamma^0\sigma^{12}\epsilon^{\mu\nu\lambda}\gamma_{\lambda}. \tag{12}$$

A particular representation is

$$\gamma^0 = I \otimes \sigma_3, \qquad \gamma^1 = \mathrm{i}\,\mathrm{diag}(1, -1) \otimes \sigma_2, \qquad \gamma^2 = \mathrm{i}I \otimes \sigma_1. \tag{13}$$

We define

$$\mathbf{a} = -\mathrm{i}\gamma^0\gamma^1\gamma^2 = -\gamma^0\sigma^{12} = \mathrm{diag}(1, -1) \otimes \sigma_3, \tag{14}$$

then the Lagrangian in (2 + 1) dimension can further be written as

$$L = \bar{\psi}\mathrm{i}\gamma^{\mu}\partial_{\mu}\psi - m\bar{\psi}\psi - (1/2)\mathbf{a}\mu_e\epsilon_{\alpha\beta\mu}\tilde{F}^{\alpha\beta}\bar{\psi}\gamma^{\mu}\psi. \tag{15}$$

By using Euler–Lagrange equation, the Dirac equation for a spin-half neutral particle with a electric dipole moment μ_e is

$$(\mathrm{i}\gamma_{\mu}\partial^{\mu} + S_{\mu}\gamma^{\mu} - m)\psi = 0, \tag{16}$$

where

$$S_{\mu} = -(1/2)\mathbf{a}\mu_e\epsilon_{\alpha\beta\mu}\tilde{F}^{\alpha\beta}. \tag{17}$$

The solution to the Dirac equation has the form

$$\psi = \mathrm{e}^{\mathrm{i}\phi_{\mathrm{HMW}}}\psi_0, \tag{18}$$

where ψ_0 is the solution for electromagnetic field free case. The phase in equation (18) is called the HMW phase, and it has the form

$$\phi_{\mathrm{HMW}} = \int^x S_{\mu}\,\mathrm{d}x^{\mu} = -\frac{1}{2}\mathbf{a}\mu_e\int^x \epsilon_{\alpha\beta\mu}\tilde{F}^{\alpha\beta}\,\mathrm{d}x^{\mu}. \tag{19}$$

The HMW phase above is the general HMW phase for a spin-1/2 neutral particle passing through an electromagnetic field. When the neutral particle moves through a pure static magnetic field, $\tilde{F}^{\mu\nu}$ reduced to $\tilde{F}^{0i}$, then we have

$$\phi_{\mathrm{HMW}} = -\mathbf{a}\mu_e\int^x \epsilon_{0ij}\tilde{F}^{0i}\,\mathrm{d}x^j = -\mathbf{a}\mu_e\int^x (\hat{k} \times \vec{B}) \cdot \mathrm{d}\vec{x}, \tag{20}$$

where $\hat{k}$ is the unit vector in z-direction and we assume that the magnetic dipole moment is always along this direction.

3. The HMW phase in noncommutative quantum mechanics

Now we are in the position to discuss the HMW topological phase in NC quantum mechanics. First let us consider the NC space case. In the noncommutative space, the coordinate and momentum operators satisfy the commutation relations (1), and the Dirac equation for the HMW effect is given by equation (2), where S_μ is given in equation (17). After the shift defined in equation (5), the Dirac equation becomes

$$\left(\mathrm{i}\gamma_\mu\partial^\mu - (1/2)\,\mathbf{a}\mu_e\epsilon_{\mu\alpha\beta}\left(\tilde{F}^{\alpha\beta} + \tfrac{1}{2}\theta^{\tau\sigma}p_\tau\partial_\sigma\tilde{F}^{\alpha\beta}\right)\gamma^\mu - m\right)\psi = 0. \tag{21}$$

This equation is defined in commuting space and the coordinate noncommutative effect appears in θ related terms. It is easy to check that the solution to this Dirac equation has the form

$$\psi = \mathrm{e}^{\mathrm{i}\hat{\phi}_{\mathrm{HMW}}}\psi_0, \tag{22}$$

where ψ_0 is the solution for electromagnetic field free case and $\hat{\phi}_{\mathrm{HMW}}$ is the HMW phase in NC space, which reads

$$\begin{aligned}\hat{\phi}_{\mathrm{HMW}} &= -\frac{1}{2}\mathbf{a}\mu_e\int^x \epsilon_{\mu\alpha\beta}\hat{\tilde{F}}^{\alpha\beta}\,\mathrm{d}x^\mu \\ &= -\frac{1}{2}\mathbf{a}\mu_e\int^x \epsilon_{\mu\alpha\beta}\tilde{F}^{\alpha\beta}\,\mathrm{d}x^\mu - \frac{1}{4}\mathbf{a}\mu_e\int^x \epsilon_{\mu\alpha\beta}\theta^{\sigma\tau}p_\sigma\partial_\tau\tilde{F}^{\alpha\beta}\,\mathrm{d}x^\mu.\end{aligned} \tag{23}$$

This is the general HMW phase for a spin-1/2 neutral particle moving in a general electromagnetic field.

Now let us consider the situation where only static electric field exists. Just like the case discussed in [17], the Hamiltonian of the particle in commuting space has the form

$$H = \frac{1}{2m}\vec{\sigma}\cdot(\vec{p} + \mathrm{i}\mu_e\vec{B})\vec{\sigma}\cdot(\vec{p} - \mathrm{i}\mu_e\vec{B}). \tag{24}$$

By using $\vec{\nabla}\cdot\vec{B} = 0$, equation (24) can be recast as

$$H = \frac{1}{2m}(\vec{p} - \vec{\mu}\times\vec{B})^2 - \frac{\mu^2B^2}{2m}, \tag{25}$$

where $\vec{\mu} = \mu_e\vec{\sigma}$, then the velocity operator can be gotten as

$$v_l = \frac{\partial H}{\partial p_l} = \frac{1}{m}[p_l - (\vec{\mu}\times\vec{B})_l]. \tag{26}$$

From this equation, we know that in noncommutative space we have

$$p_l = mv_l + (\vec{\mu}\times\vec{B})_l + \mathcal{O}(\theta). \tag{27}$$

Insert equation (27) into (23) and note that

$$\tilde{F}^{\alpha\beta} \longrightarrow \tilde{F}^{0i} \qquad \text{and} \qquad \theta^{ij} = \theta\epsilon^{ij}, \qquad \theta^{0\mu} = \theta^{\mu0} = 0, \tag{28}$$

we have

$$\hat{\phi}_{\mathrm{HMW}} = \phi_{\mathrm{HMW}} + \delta\phi_{\mathrm{NCS}}, \tag{29}$$

where ϕ_{HMW} is the HMW phase in commuting space given by (20), the added phase $\delta\phi_{\mathrm{NCS}}$, related to the noncommutativity of space, is given by

$$\begin{aligned}\delta\phi_{\mathrm{NCS}} &= -\frac{1}{2}\mathbf{a}\mu_e\int^x \epsilon_{\mu0i}\theta\epsilon^{\alpha\beta}[mv_\alpha + (\vec{\mu}\times\vec{B})_\alpha]\partial_\beta\tilde{F}^{0i}\mathrm{d}x^\mu \\ &= \frac{1}{2}\mathbf{a}\mu_e\theta\epsilon^{ij}\int^x [k_j + (\vec{\mu}\times\vec{B})_j](\partial_iB^2\,\mathrm{d}x^1 - \partial_iB^1\,\mathrm{d}x^2),\end{aligned} \tag{30}$$

where $k_j = mv_j$, and the result here coincides with the result given in [18], where the tedious star product calculation has been used. The first term in equation (28) is a velocity-dependent correction insensitive to the topology of the manifold and could modify the phase shift, the second term is a noncommutative correction to the vortex which does not contribute to the line spectrum.

When both space–space and momentum–momentum non-commutating are considered, i.e. we study the problem on NC phase space, the Dirac equation for the HMW model is the same as the case on NC space, but the star product and the shifts are defined in equations (7) and (9). After a similar procedure as in NC space, we got the Dirac equation on NC phase space as

$$\left\{-\gamma^\mu p_\mu - \frac{1}{2\alpha^2}\gamma^\mu\bar{\theta}_{\mu\nu}x^\nu - (1/2)\mathbf{a}\mu_e\epsilon_{\mu\alpha\beta}\left[\tilde{F}^{\alpha\beta} + \frac{1}{2\alpha^2}\theta^{\tau\sigma}p_\tau\partial_\sigma\tilde{F}^{\alpha\beta}\right]\gamma^\mu - m'\right\}\psi = 0, \tag{31}$$

where $m' = m/\alpha$. The solution to (31) is

$$\psi = \mathrm{e}^{\mathrm{i}\hat{\varphi}_{\mathrm{HMW}}}\psi_0, \tag{32}$$

where ψ_0 is the solution of the Dirac equation for free particle with mass m' and $\hat{\varphi}_{\mathrm{HMW}}$ stands for the HMW phase in NC phase space, and it has the following form:

$$\hat{\varphi}_{\mathrm{HMW}} = -\frac{1}{2}\mathbf{a}\mu_e\int^x\epsilon_{\mu\alpha\beta}\tilde{F}^{\alpha\beta}\,\mathrm{d}x^\mu - \frac{1}{2\alpha^2}\int^x\bar{\theta}_{ij}x_j\,\mathrm{d}x_i - \frac{1}{4\alpha^2}\mathbf{a}\mu_e\int^x\epsilon_{\mu\alpha\beta}\theta^{\sigma\tau}p_\sigma\partial_\tau\tilde{F}^{\alpha\beta}\,\mathrm{d}x^\mu. \tag{33}$$

Equation (33) is the general HMW phase in noncommutative phase space. Once again for this case only static magnetic field exists, then the HMW phase reduces to

$$\hat{\varphi}_{\mathrm{HMW}} = \phi_{\mathrm{HMW}} + \delta\phi_{\mathrm{NCPS}}, \tag{34}$$

where

$$\begin{aligned}\delta\phi_{\mathrm{NCPS}} &= -\frac{1}{2\alpha^2}\int^x\bar{\theta}_{ij}x_j\,\mathrm{d}x_i - \frac{1}{2\alpha^2}\mathbf{a}\mu_e\int^x\epsilon_{\mu0i}\theta\epsilon^{\alpha\beta}[m'v_\alpha + (\vec{\mu}\times\vec{B})_\alpha]\partial_\beta\tilde{F}^{0i}\,\mathrm{d}x^\mu\\ &= -\frac{1}{2\alpha^2}\int^x\bar{\theta}_{ij}x_j\,\mathrm{d}x_i + \frac{1}{2\alpha^2}\mathbf{a}\mu_e\theta\epsilon^{ij}\int^x[k'_j + (\vec{\mu}\times\vec{B})_j](\partial_i B^2\,\mathrm{d}x^1 - \partial_i B^1\,\mathrm{d}x^2),\end{aligned} \tag{35}$$

in which $k'_j = m'v_j$, $p_l = m'v_l + (\vec{\mu}\times\vec{B})_l + \mathcal{O}(\theta)$ have been applied and we omit the second-order terms in θ. The term $\delta\phi_{\mathrm{NCPS}}$ represents the noncommutativity for both space and momentum. The first term in $\delta\phi_{\mathrm{NCPS}}$ is a contribution purely from the noncommutativity of the momenta, similar to equation (30), the second term is a velocity-dependent correction and the third term is a correction to the vortex of magnetic field. In two-dimensional noncommutative plane, $\bar{\theta}_{ij} = \bar{\theta}\epsilon_{ij}$, and the two NC parameters θ and $\bar{\theta}$ are related by $\bar{\theta} = 4\alpha^2(1-\alpha^2)/\theta$ [6]. When $\alpha = 1$, which will lead to $\bar{\theta}_{ij} = 0$, $\delta\phi_{\mathrm{NCPS}}$ returns to $\delta\phi_{\mathrm{NCS}}$, namely, the HMW phase on NC phase space will return to the HMW phase in NC space.

4. Conclusion remarks

In this paper, the HMW effect is studied on both noncommutative space and noncommutative phase space. Instead of doing tedious star product calculation, we use the 'shift' method, i.e. the star product in the Dirac equation can be replaced by the shift defined in [2] and together with the shift we defined in (5) for NC space and (9) for NC phase space. These shifts are exact equivalent to the star product. The additional HMW phase terms (23) and (30) in NC space and the terms (33) and (35) in NC phase space are new results of our paper, these two term are related to noncommutativity of space and phase space. This effect is expected to be tested at a very high energy level, and the experimental observation of the effect remains to be further studied.

The method we use in this paper may also be employed to other physics problem on NC space and NC phase space. The further study on the issue will be reported in our forthcoming papers.

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